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# UK2070 Futures

# Post-COVID Scenario Modelling

## Technical Report
## Part3: Modelling Appendix

**(Refer Part 1 for Report)**
**(Refer Part 2 for Maps Annex)**

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# The LUISA Model and Its Application to the UK2070 Futures Modelling Study

This model appendix is organized as follows. Section A1 introduces the formal structure of the LUISA2.3 model. Section A2 discusses the model solving algorithm in a step-by-step manner. Section A3 summarizes the zoning system in the model. Lists of model variables and behavioural parameters are provided in Section A4. Further technical details are available from the Martin Centre team at University of Cambridge.

## A1 Structure of the LUISA2.02 Model

Suppose that the city region is divided into $\Im$ core zones plus $\wp$ peripheral zones. Core zones represent the core study area where detailed policy analyses are conducted with relatively fine spatial granularity; while the peripheral zones represent the wider region outside the core study area which exchanges production factors (e.g. labour) and trades goods & services with the core zones. $\mathbb{N} = \Im + \wp$ thus denotes all modelled zones. Each of the model zones has $r = 1, \dots, \mathcal{R}$ basic industries and $f = 1, \dots, F$ consumer types. Table 21 summarizes the model segmentations in the model.

TABLE 21 SEGMENTATIONS IN THE MODEL

| | Industry types | Consumer types | Residential floorspace types | Commercial floorspace types |
|---|---|---|---|---|
| Core zones | $r = 1, \dots, \mathcal{R}$ | $f = 1, \dots, F$ | $m = 1, \dots, \aleph_1$ | $k = 1, \dots, \aleph_2$ |
| Peripheral zones | $r = 1, \dots, \mathcal{R}$ | $f = 1, \dots, F$ | $m = 1, \dots, \aleph_1$ | $k = 1, \dots, \aleph_2$ |

We introduce the following model components in turn: producers, final consumers, location choices, stock constraints and equilibrium conditions.

### A1.1 Producers

The producers are represented by a set of production functions that define how they use capital, labour, floorspace and intermediate inputs (raw materials and services). A nested Cobb-Douglas CES (CD-CES) function has been broadly accepted as a standard for this purpose in spatial general equilibrium analyses since Krugman (1991) and Fujita et al. (1999). We follow Anas and Liu (2007) and Jin et al. (2013), and define the production function as a variant of the CD-CES specification.

$$X_{rj} = E_{rj} A_{rj} (K_r)^{\nu_r} \left(\sum_f \kappa_{rfj} L_{fj}^{\theta_r}\right)^{\frac{\delta_r}{\theta_r}} \left(\sum_k \chi_{rkj} B_{kj}^{\zeta_r}\right)^{\frac{\mu_r}{\zeta_r}} \prod_s (Y_{rsj})^{\gamma_{rs}} \quad (1)$$

where $X_{rj}$ is the production output of industry $r$ in zone $j$; $K_r$, $L_{fj}$, $B_{kj}$ and $Y_{rsj}$ are the capital, labour, business floorspace and intermediate input, respectively; $\nu_r$, $\delta_r$, $\mu_r$ and $\gamma_{rs}$ are cost share parameters for the respective input group. This function is Cobb-Douglas and is constant returns to scale by $\nu_r + \delta_r + \mu_r + \sum_s \gamma_{rs} = 1$. The elasticity of substitution between any two labour and building floorspace varieties is $1/(1-\theta_r)$ and $1/(1-\zeta_r)$, respectively. $\kappa_{rfj}, \chi_{rkj} \geq 0$ are input-specific constants for labour and business floorspace varieties, respectively. These constants allow us to specify input-specific preference within each input bundle. $A_{rj}$ is a function of the economic mass for industry $r$ in zone $j$ that represents Hicksian-neutral Total Factor Productivity (TFP) effects resulting from learning and transfer of tacit knowledge (Graham & Kim, 2008; Rice, Venables, & Patacchini, 2006), which is an important component of urban agglomeration effects. $E_{rj}$ is a constant scalar representing any additional zonal effects on total factor productivity. We define $A_{rj} = \underline{A}_{rj}(M_j/\underline{M}_j)^{\pi}$, where $\underline{A}_{rj}$ is a constant representing the baseline agglomeration effects, $M_j$ is a function of the economic mass of zone $j$, $\underline{M}_j$ is a constant representing the baseline economic mass in $j$; $\pi$ is a scale parameter. The function of economic mass builds on the concept of effective density (Graham, Gibbons, & Martin, 2009).

$$M_j = \sum_f \sum_i \frac{L_{fi}}{\chi_{fij}} \quad (2)$$

where $L_{fi}$ is the total size of labour type $f$ in zone $i$ (including zone $j$) that is relevant to production zone $j$, and $\chi_{fij}$ is the travel time from location $i$ to $j$ for labour type $f$.

We assume that each firm minimizes the cost subject to the production demand and the price of each input variety. The *conditional input demand* (given target output $X_{rj}$) of each input factor can be derived as follows:

$$K_r = \frac{1}{\rho} \nu_r p_{rj} X_{rj} \tag{3}$$

$$L_{rfj} = \frac{\kappa_{rfj}^{\frac{1}{1-\theta_r}} w_{fj}^{\frac{1}{\theta_r - 1}}}{\sum_s \kappa_{rsj}^{\frac{1}{1-\theta_r}} w_{sj}^{\frac{\theta_f}{\theta_r - 1}}} \delta_r p_{rj} X_{rj} \tag{4}$$

$$B_{rkj} = \frac{\chi_{rkj}^{\frac{1}{1-\zeta_r}} R_{kj}^{\frac{1}{\zeta_r - 1}}}{\sum_s \chi_{rsj}^{\frac{1}{1-\zeta_r}} R_{sj}^{\frac{\zeta_r}{\zeta_r - 1}}} \mu_r p_{rj} X_{rj} \tag{5}$$

$$Y_{rsj} = \frac{\gamma_{rs} p_{rj} X_{rj}}{p_{rs|j}^*} \tag{6}$$

where $p_{rj}$ is the unit production price of industry $r$ in zone $j$; $\rho$ is the exogenous price of business capital (i.e. the real interest rate); $w_{fj}$ is the hourly wage of labour type $f$; $R_{kj}$ is the average rent for business floorspace type $k$; and $p_{rs|j}^*$ is the average delivered price of intermediate input type $s$ for producing product type $r$ in zone $j$.
The minimized production price can then be calculated by substituting the above conditional demands into the production function. As zero profit is assumed at any level of output, the minimized price equals the average and the marginal cost, which takes the form:

$$p_{rj} = \frac{\rho^{\nu_r} \left( \sum_f \kappa_{rfj}^{\frac{1}{1-\theta_r}} w_{fj}^{\frac{\theta_r}{\theta_r - 1}} \right)^{\frac{\delta_r \theta_r - 1}{\theta_r}} \left( \sum_k \chi_{rkj}^{\frac{1}{1-\zeta_r}} R_{kj}^{\frac{\zeta_r}{\zeta_r - 1}} \right)^{\frac{\mu_r \zeta_r - 1}{\zeta_r}} \prod_m p_{rs|j}^{*\ \gamma_{rs}}}{E_{rj} A_j \nu_r^{\nu_r} \delta_r^{\ \delta_r} \mu_r^{\mu_r} \prod_s \gamma_{rs}^{\gamma_{rs}}} \tag{7}$$

## A1.2 Final Consumers

Final consumers are categorized into $f = 1, \dots, F$ types according to their employment status and socio-economic level. $H_f$ is the exogenous number of consumers in group $f$. Consumers in socio-economic group $f$ receive both wage and nonwage income, except group $f = F$ denoting the non-employed consumers who do not have wage income but receive nonwage income through social welfare transfer. The wage income is modelled endogenously subject to equilibrium conditions, while the nonwage income is subject to the *a priori* welfare transfer scheme.
Each consumer makes a set of discrete and continuous choices. For discrete choices, the employed residents decide where to work and where to live jointly from $j = 1, \dots, \mathbb{N}$ employment zones and $i = 1, \dots, \mathbb{N}$ residence zones; the non-employed residents choose their residence location from $i = 1, \dots, \mathbb{N}$ residence zones. Both the employed and non-employed consumers choose where to source goods & services from $z = 1, \dots, \mathbb{N}$ production zones. The remaining choices entail continues variables and are conditional on the above discrete location choices. Consumers then decide on: 1) the annual consumption of each goods & services variety; 2) the quantity of type $m$ housing floorspace to rent; 3) the use of time between work and leisure in the case of employed consumers. All consumers are assumed to maximize their utility from the mixed discrete-continuous choice.

Following the random utility framework (McFadden, 1973), the utility of consumer type $f$ living in zone $i$ and working in zone $j$ takes the form $U_{fij}^* = U_{fij} + e_{fij}$ where $U_{fij}$ is the observable quantity-based utility and $e_{fij}$ is the error term which measures the unobservable utility variance among consumers. The observable utility $U_{fij}$ is given by:

$$U_{fij} = \alpha_f \ln\left(\sum_r \sum_z \xi_{rfz}\left(Z_{rz|fij}\right)^{\eta_f}\right)^{\frac{1}{\eta_f}} + \beta_f \ln\left(\sum_m \iota_{mfi}\left(b_{m|fij}\right)^{\sigma_f}\right)^{\frac{1}{\sigma_f}} + \gamma_f \ln l_{fij} \quad (8)$$

$$\textit{subject to budget constraint}: \sum_{r,z} \left(p_{rz} + c_f 2 g_{fiz}\right) Z_{rz|fij} + \sum_m r_{mi} b_{m|fij} + \Delta_f 2 D g_{fij}$$

$$= \Delta_f w_{fj}\left(N - 2DG_{fij} - \sum_{r,z} c_f Z_{rz|fij} 2G_{fiz} - l_{fij}\right) + \mathcal{M}_{fi}$$

$$\textit{and time constraint}: \quad N - \sum_{r,z} c_f Z_{rz|fij} 2G_{fiz} - \Delta_f\left(l_{fij} + 2DG_{fij}\right) \geq 0$$

In equation (8), we assume Cobb-Douglas preference between goods & services $Z_{rz|fij}$, housing $b_{m|fij}$ and leisure time $l_{fij}$. $\alpha_f + \beta_f + \gamma_f = 1$ are the expenditure coefficients for each consumption bundle. The varieties of goods & services and housing are assumed to be imperfect substitutes (Dixit & Stiglitz, 1977), and the elasticity of substitution is governed by $\eta_f$ and $\sigma_f$ for goods & services and housing, respectively. $\xi_{rfz}, \iota_{mfi} > 0$ are the input-specific constants measuring the inherent attractiveness of the goods & services, and housing varieties for consumers type $f$, which is calibrated empirically.

For the budget constraint in equation (8), the right-hand side of the function is the total income and the left-hand side is the total expenditure. Specifically, $p_{rz}$ is the mill price for goods & services type $r$ produced in zone $z$; $g_{fiz}$ and $G_{fiz}$ is the expected one-way monetary cost and travel time from $i$ to $z$ for customers type $f$, respectively[2]; $c_f$ is an exogenous coefficient that measures the cost for delivering a unit of goods & services as percentage of the normal trip cost. $r_{mi}$ is the housing rent of type $m$ in zone $i$; $w_{fj}$ is the hourly wage rate for labour type $f$ working in zone $j$. $\Delta_f$ is the employment status of the consumer type $f$. For all employed consumers $\Delta_f = 1$; otherwise $\Delta_f = 0$. $\mathcal{M}_{fi}$ is the nonwage income of consumer type $f$ in zone $i$. It consists of normal investment returns on real estate in the city region (endogenous in the model) as well as the individual share of social welfare transfer and amenity gains (subject to *a priori* scheme). As for the time constraint, $D$ is the exogenous number of working days per annum; $N = 24D$ is the exoge*n*ous total annual time endowment. For the non-employed consumers ($\Delta_f = 0$), the model only accounts for the time for shopping, as they do not commute and have zero value of time for leisure time.

We can rewrite the budget constraint in equation (8) to consider the value of time for shopping travel as a part of the delivered price. The new constraint function is equivalent to equation (8).

$$\sum_{r,z} p^*_{rz|fij} Z_{rz|fij} + \sum_m r_{mi} b_{m|fi} + \Delta_f 2 D g_{ij} \quad (9)$$
$$= \Delta_f w_{fj}\left(N - 2DG_{ij} - l_{fij}\right) + \mathcal{M}_{fi}$$

where $p^*_{rz|fij}$ is the full delivered price of a unit of goods & services type $r$ produced in zone $z$ purchased by consumer type $f$ living in zone $i$ and working in zone $j$. We use the subscript $z$ to denote the production location of goods & services and $j$ as the employment location for employed workers. The full delivered price for final consumers $p^*_{rz|fij}$ is given by:

$$p^*_{rz|fij} = p_{rz} + c_f 2\left(g_{iz} + \Delta_f G_{iz} w_{fj}\right) \quad (10)$$

Accordingly, the full disposable income of the consumer type $(fij)$ net of commuting costs is given by:

$$\Omega_{fij} = \Delta_f w_{fj}\left(N - 2DG_{ij} - l_{fij}\right) - \Delta_f 2 D g_{ij} + \mathcal{M}_{fi} \quad (11)$$

Under the above budget and time constraint, we can then derive the *Marshallian* demand for goods & services, housing and leisure time in Eq. 3.12, Eq. 3.13 and Eq. 3.14, respectively.

[2] The monetary cost and travel time is composite over all available travel modes. For the moment, we do not consider the time-of-day and purpose variations in travel time and cost.

$$\bar{Z}_{r|fij} = \frac{\xi_{rfz}^{\frac{1}{1-\eta_f}} \bar{p}_{r|fij}^{\frac{1}{\eta_f - 1}}}{\sum_s \xi_{rfz}^{\frac{1}{1-\eta_f}} \bar{p}_{s|fij}^{\frac{\eta_f}{\eta_f - 1}}} \alpha_f \Omega_{fij} \tag{12}$$

$$b_{m|fij} = \frac{\iota_{mfi}^{\frac{1}{1-\sigma_f}} r_{mi}^{\frac{1}{\sigma_f - 1}}}{\sum_s \iota_{si}^{\frac{1}{1-\sigma_f}} r_{si}^{\frac{\sigma_f}{\sigma_f - 1}}} \beta_f \Omega_{fij} \tag{13}$$

$$l_{fij} = \frac{\gamma_f \Omega_{fij}}{w_{fj}} \tag{14}$$

where $\bar{Z}_{r|fij}$ is the aggregate demand for product type $r$ for consumer type $(fij)$; $\bar{p}_{r|fij}$ is the probability-weighted average price of product type $r$ faced by consumer type $(fij)$. The formulation of $\bar{p}_{r|fij}$ and $\bar{Z}_{r|fij}$ and the associated discrete-choice probability function will be introduced shortly.

In addition to the *Marshallian* utility function (maximizing utility subject to budget constraints), which is used in base-year model calibration, the model employs the *Hicksian* utility function in forecasts. The Hicksian utility function differs from the Marshallian utility function in that it minimizes the expenditure given fixed utility. The use of Hicksian utility function in forecast mode implies that consumers are assumed to maintain, if not increase, their base-year utility level in future years by altering their locational and consumption choices. Under the same Nested-CES configuration and parameterization, the Marshallian and Hicksian utility functions are consistent in base-year model calibration, in the sense that the derived Marshallian demands (given observed budget constraint) are identical to the Hicksian demands (given the Marshallian utility). In forecast mode, the Hicksian utility function will replace the Marshallian utility function. The implication is that consumers will have to raise the income if the cost of living (i.e. prices of goods & services and housing rents) goes up, in order to maintain the same utility level. The need for increasing income will then be represented by an upward pressure on labour wage. In case the cost of living goes down (e.g. abundance of housing supply), the model assumes that the local wage level would not decrease subject to global price adjustment. Nonetheless the resulting extra utility gain will be competed out in spatial equilibrium as more residents move into the area, which in turn drives up the cost of living. For the Hicksian utility function, the minimized expenditure given the utility $U_{fij}$ is defined as:

$$\Omega_{fij}^{Hicksian} = \alpha_f^{-\alpha_f} \beta_f^{-\beta_f} \gamma_f^{-\gamma_f} \left[ \left( \sum_r \sum_z \xi_{rfz}^{\frac{1}{1-\eta_f}} \bar{p}_{r|fij}^{\frac{\eta_f}{\eta_f - 1}} \right)^{\frac{\eta_{f-1}}{\eta_f}} \right]^{\alpha_f} \left[ \left( \sum_m \iota_{mfi}^{\frac{1}{1-\sigma_f}} r_{mi}^{\frac{\sigma_f}{\sigma_f - 1}} \right)^{\frac{\sigma_f - 1}{\sigma_f}} \right]^{\beta_f} (w_{fj})^{\gamma_f} U_{fij} \tag{15}$$

The total annual labour working time $N_{fij}$ for the employed consumer type $(fij)$ is thus determined by subtracting the total travel time for commuting and shopping, and the annual leisure time from the annual time endowment $N$.

$$N_{fij} = N - 2DG_{ij} - \sum_{r,z} c_f Z_{rz|fij} 2G_{iz} - l_{fij} \geq 0 \tag{16}$$

The next step is to evaluate the direct utility function (8) to get the price-based indirect utility function $\widetilde{U}_{fij}$, which is given by:

$$\widetilde{U}_{fij} = \ln \Omega_{fij} - \alpha_f \frac{\eta_f - 1}{\eta_f} \ln \left( \sum_r \sum_z \xi_{rfz}^{\frac{1}{1-\eta_f}} \bar{p}_{r|fij}^{\frac{\eta_f}{\eta_f - 1}} \right) - \beta_f \frac{\sigma_f - 1}{\sigma_f} \ln \left( \sum_m \iota_{mfi}^{\frac{1}{1-\sigma_f}} r_{mi}^{\frac{\sigma_f}{\sigma_f - 1}} \right) - \gamma_f \ln w_{fj} \tag{17}$$

Note that the quantity-based and the price-based utility functions are mathematically equivalent in static equilibrium. However, for the purpose of welfare evaluation over time, particularly in long-term forecast that involves macroeconomic changes (e.g. price-level changes due to growth, inflation or deflation), the quantity-based direct utility function offers a more intuitive and straightforward measure than the price-based counterpart. Therefore, we use the price-based utility in static equilibria and the quantity-based utility for welfare analysis.

### A1.3 Location Choices

The location choices in the model include: 1) sourcing goods & services for final consumers; 2) the employment-residence choice (or residence location choice if employment is exogenous) for the employed residents. Both location choices are modelled in the spatial equilibrium framework. Another important aspect of location choice modelling is the articulation of travel disutility. We summarize the measure of travel disutility in the model by the end of this section.

#### A1.3.1 Sourcing goods and services

In the model, consumers do not only decide the quantity of each product to purchase, but also where to source them. The former decision is based on average delivered price of each product thus is continuous in nature; while the latter choice is discrete involving limited number of location alternatives. We represent this mixed discrete-continuous choice problem by combining two different choice models. For the continuous choice on quantities, a nested CES function is applied to consider the substitution effects within the consumption bundle. For the discrete location choice, the sourcing pattern is modelled with a multinomial logit probabilistic model. The probability of obtaining product type $r$ from zone $z$ to consumer type $f$ living in zone $i$ (and working in zone $j$, if employed) is given by:

$$P_{rz|fij} = \frac{S_z \exp\left(-\lambda_{f|r}(p_{rz} + c_f \chi_{fiz} + \psi_{riz} - E_{rfz})\right)}{\sum_n S_n \exp\left(-\lambda_{f|r}(p_{rn} + c_f \chi_{fin} + \psi_{rin} - E_{rfn})\right)} \tag{18}$$

where $S_z$ is a size term that corrects for the bias introduced by the uneven sizes of zones in the model (Ben-Akiva & Lerman, 1985); $\lambda_{f|r}$ is the dispersion parameter. $c_f$ is a coefficient measuring the cost for delivering a unit of goods & services as percentage of normal trip cost; $\chi_{fiz}$ is a travel disutility function; $\psi_{riz}$ are observable non-monetary barriers for trading between zone $i$ and zone $z$; $E_{rfz}$ is the residual attractiveness term which is calibrated empirically. In the model, consumers will shop to all potential production zones, rather than the zone with the cheapest delivered price only[3]. A similar probability function can be applied to model the sourcing of intermediate inputs for producers.

With the above probability, we can derive the weighted average price of product type $r$ faced by consumer type $(fij)$. Note that this weighted average price considers the consumption inputs from all possible production locations, thus the dimension is $[r]$.

$$\bar{p}_{r|fij} = \sum_z p^*_{rz|fij} P_{rz|fij} \tag{19}$$

where $p^*_{rz|fij}$ is the full delivered price including the value of time for travel. The purpose of deriving $\bar{p}_{r|fij}$ is to link the discrete location choice with the continuous choice of consumption quantities. For residents living in zone $i$, they first choose how much to consume for each product type $(\bar{Z}_{r|fij})$, regardless of the their production locations. This continuous choice is made based on the weighted average price $\bar{p}_{r|fij}$ through CES functions. The discrete-choice probability in Eq. 3.17 then distributes the aggregate demand $\bar{Z}_{r|fij}$ to each production location $z$. This distribution process is given by:

$$Z_{rz|fij} = P_{rz|fij} \bar{Z}_{r|fij} \tag{20}$$

This function is used to derive the total production demand for product type $r$ in zone $z$.

#### A1.3.2 Employment/residence location choice

In the model, we differentiate the location choice of employed residents and the non-employed. For employed residents we assume that they respond quickly to the utility changes and are mobile in terms of employment-residence relocation in static equilibria. By contrast, the relocation of non-employed residents is inertia-prone, i.e. there may be a lag of many years between a utility change and household relocation. We thus deal the relocation of non-employed households outside the equilibrium framework through recursive dynamic model or model assumptions. This section first introduces the discrete choice model for employment-residence

[3] By "shop" we refer to any non-work trip that involves the purchase of goods and services. We ignore trip chains and travels that do not originate from home.

joint choice. The residence location choice model as an abridged version the former model will be discussed afterwards.
For the employment-residence choice of employed residents, a multinomial logit model is developed. The probability of consumer $f$ working in zone $j$ choosing to live in zone $i$ is defined as:

$$P_{fij} = \frac{S_{ij}\exp\left(\lambda_f v_{fij}\right)}{\sum_{m,n} S_{mn}\exp\left(\lambda_{fI} v_{fmn}\right)} \tag{21}$$

where

$$v_{fij} = \widetilde{U}_{fij} - d_{fij} + \psi_{fij} + E_{fij} + e_{fij} \tag{22}$$

$S_{ij}$ is the a size term that addresses the size of residence/employment opportunities in zone $i/j$; $\lambda_{f|I}$ is the dispersion parameter; $\widetilde{U}_{fij}$ is the consumption utility of consumer $f$ living in zone $i$ and working in zone $j$; $d_{fij}$ is the travel disutility of travelling from zone $i$ to $j$; $E_{fij}$ is the residual attractiveness of location pair $(i,j)$, and $e_{fij}$ is the unobserved error term.
For the residence choice of employed residents, the probability of consumer $f$ choosing to live in zone $i$, given the employment location $j$, is defined as:

$$P_{fi|j} = \frac{S_i\exp\left(\lambda_{f|I} v_{fi|j}\right)}{\sum_m S_m\exp\left(\lambda_{f|I} v_{fm|j}\right)} \tag{23}$$

where

$$v_{fi|j} = \widetilde{U}_{fi|j} - d_{fi|j} + \psi_{fi|j} + E_{fi|j} + e_{fi|j} \tag{24}$$

$v_{fi|j}$ is the residence location utility of zone $i$ for resident type $f$, given the chosen workplace $j$; $\lambda_{f|I}$ is the dispersion parameter. The other variables follow the same definitions as in function $v_{fij}$, except that the employment location $j$ is given.

A1.3.3 Travel disutility

In the model, the $\chi_{fij}$ function is introduced to represent the attributes of travel for traveller type $f$ from $i$ to $j$. We differentiate the $\chi_{fij}$ function for different uses throughout the model. In this section, we summarize the use of the $\chi_{fij}$ function. For measuring the economic mass (as in Eq. 2), we define $\chi_{fij} = 2G_{fiz}$, which is the round-trip travel time (in hourly term) between zone $i$ and $j$ for traveller type $f$.
For sourcing goods & services (as in Eq. 18), we define $\chi_{fiz} = 2(g_{fiz}/\varsigma_f\overline{w}_{fi} + G_{fiz})$, where $\overline{w}_{fi}$ is the average hourly wage of type-$f$ employed residents living in zone $i$[4], and $\varsigma_f \in (0,1]$ is a decay coefficient, implying that the shopping trip being partly voluntary thus its value of time is not fully valued by the traveller. The front multiplier transforms the one-way cost into round-trip cost (de Dios OrtÃozar & Willumsen, 2011). The above formulation adopts the time unit (hour), and considers both the travel time and the monetary cost. The monetary cost is transformed into time unit by dividing it by the value of time $\varsigma_f\overline{w}_{fi}$. Note that this time-based travel disutility is only used for modelling location choices. The actual transport costs, including the value of time, are measured in monetary unit in the equilibrating process.
For the employment-residence location choice, it is important to consider the realistic commuting patterns within a large city region. City regions with reasonably self-contained commuting catchment today tend to have a radius of 50km or more. At this metropolitan scale, extensive analyses of travel choices data show that a $d_{ij}$ function (as in Eq. 22) that is linear to travel costs and times will have great difficulties in representing realistic demand elasticity throughout (Jin et al., 2013); a non-linear transformation of utilities is required (Gaudry & Laferriére, 1989). Fox et al (2009) devise a log-linear transformation that is a close equivalent to the Box-Cox function whilst being easier to calibrate. This function is given by:

$$d_{fij} = a_{f|d}\chi_{fij} + \left(1 - a_{f|d}\right)\ln\chi_{fij} - a_{f|d} \tag{25}$$

where $\chi_{fij} = 2DG_{fiz}$, i.e. the annual total commuting time between zone $i$ and $j$ for labour type $f$, and $a_{f|d}$ is a log-linear parameter. The reason why we do not account for the monetary cost is

[4] To distinguish $\overline{w}_{fi}$ and $w_{fj}$, the latter is the hourly wage of labour type $f$ at production zone $j$, while the former is the average wage for labour type-$f$ living in residence zone $i$, weighted by the modelled labour distribution to all employment locations.

that the monetary cost is already accounted for in the consumption utility function (see the budget constraint in Eq. 8). To avoid double counting, we thus only consider the travel time in the $\chi_{fij}$ function.

To demonstrate the non-linear feature of the above function, we plot the log-linear travel disutility versus the linear counterpart in **Error! Reference source not found.**. It shows that the modelled elasticity of the log-linear function varies for different distance ranges. Specifically, the elasticity of disutility with regard to distance is higher for short-distance range (approx. 0-15 km), and becomes lower for long-distance range (approx. > 15 km).

### A1.4 Stock Constraints

We define stock constraints to cover land/floorspace and transport infrastructure which may evolve or “churn” slowly. In the model, the stock constraints include: 1) the zonal supply of housing floorspace varieties ($\hat{b}_{mi}$) and business floorspace varieties ($\hat{B}_{ki}$); 2) the expected transport monetary cost ($g_{fij}$) and travel time ($G_{fij}$) for consumer type $f$; 3) the zonal number of non-employed residents ($H_F$).

In the model, such stock constraints remain exogenous for any static period and will be updated periodically in a non-equilibrium manner. The underlying assumption is that land/floorspace and transport infrastructure respond to demand slowly and indivisibly, subject to regulation, planning, construction, commission and decommission (Jin et al., 2013). User-defined supply scenarios are likely to be the most appropriate in order to reflect policy targets and background changes. As for the relocation of non-employed residents, it is assumed that there is a time lag between a utility change and household relocation.

### A1.5 Equilibrium Conditions

The general equilibrium structure of the model requires three sets of equilibrium conditions to be satisfied simultaneously, conditional on the transport conditions $\boldsymbol{g}$ and $\boldsymbol{G}$.

1) All consumers maximize utility subject to budget and time constraint, or minimise expenditure subject to given utility target.
2) All producers minimize cost subject to supply constraint of input factors and technology. Producers are competitive and operate under constant returns to scale. The minimized production price equals the average and marginal cost, implying zero economic profit.
3) All markets clear with zero excess demands. This applies to: a) the residential and business floorspace markets; b) the labour market for each socio-economic group at each production zone; c) the product market of each product type at each production zone.

The above equilibrium conditions are formulated in the model as follows:

#### A1.5.1 Product markets

The market clearance condition in both zonal and regional product markets prescribes that in each of the $j = 1, \ldots, \mathbb{N}$ production zone, the production output of each industry should equal the total production demand plus net export. Let $Y_{rj|sn}$ be the intermediate demand for industry $r$ in zone $j$ for producing product $s$ in zone $n$ and $\Xi_{rj}$ be the exogenous net export for industry $r$ in zone $j$. The zero excess demands in product markets require:

$$\sum_{f,z} H_{fi} P_{rz|fij} \bar{z}_{r|fij} + \sum_{s,n} Y_{rj|sn} + \Xi_{rj} = X_{rj} \tag{26}$$

#### A1.5.2 Labour Markets

In each of the $j = 1, \ldots, \mathbb{N}$ production zone, the annual labour demand in hourly term for each of the $f = 1, \ldots, F-1$ labour group must equal the working hours supplied by the respective labour group, net of the time for commuting, shopping and leisure.

$$\sum_{r} L_{rfj} = \sum_{i} H_{fi}\, P_{fij} \left( N - 2DG_{fij} - \sum_{r,z} c_f Z_{rz|fij} 2G_{fiz} - l_{fij} \right) \tag{27}$$

#### A1.5.3 Floorspace Markets

We treat the zonal building floorspace as exogenous supply constraints in static equilibria, and update them through Recursive Dynamic models. The market clearance in floorspace markets requires that in static equilibrium, the zonal demand for each type of residential and business floorspace must equal the corresponding zonal supply constraint.

$$\sum_{f,j} b_{m|fij} = \hat{b}_{mi} \quad (28)$$

$$\sum_{r} B_{rkj} = \hat{B}_{kj} \quad (29)$$

where $\hat{b}_{mi}$ and $\hat{B}_{kj}$ is the zonal supply constraint for housing and business floorspace, respectively.

As a summary, the aforementioned equilibrium conditions define the aggregate behavioural rules of agents, and specify how they interact with each other in respective market. In fact, the equilibrium conditions constitute the economic foundation of general equilibrium models, and it is a theoretical necessity to satisfy such conditions in equilibrium analysis.

## A2 Model Algorithm

In the previous section, we present the formal structure of the Spatial Equilibrium model. Given the exogenous stock constraints (building floorspace supply, transport infrastructure and non-employed households), the aforementioned equations and variables complete the spatial general equilibrium of the model. Following the convention of spatial equilibrium models, we solve the static equilibrium in a sequential manner, which is specified in FIGURE 22.

The solving algorithm for the Spatial Equilibrium model is as follows:

*STEP 0 (Initialization).* Arbitrary exogenous vectors of rents $(\boldsymbol{R}, \boldsymbol{r})$, wages $(\boldsymbol{w})$ serve as initial inputs. Given the guessed values, as well as the given transport conditions $\boldsymbol{G}$ and $\boldsymbol{g}$ and all parameters, the following sequentially arranged steps complete a single iteration of the SE model.

*STEP 1 (Production prices).* The zero economic profit equation (7) is solved for the equilibrium production price $\boldsymbol{p}$, given wages $\mathbf{w}$ and business floorspace rents $\mathbf{R}$.

*STEP 2 (Location choices).* Residents make discrete location choice for sourcing goods & services with equation (18). Employed residents make joint location choices with Equation 21 or 23.

*STEP 3 (Outputs).* Given the production price $\boldsymbol{p}$ from STEP 1 and the location choices from STEP 2, the final demand for production $\boldsymbol{F}$ can be solved with the *Marshallian* demand function (12) and the zero-excess-demand equation (26). The total production demand $\boldsymbol{X}$, including the intermediate demand, can be derived with the classical input-output solution $\boldsymbol{X} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{F}$, where $\boldsymbol{A} = [\gamma_{rs}]$ is the matrix of input-output coefficients.

*STEP 4 (Rents).* Given the production price $\boldsymbol{p}$ from STEP 1 and the production outputs $\boldsymbol{X}$ from STEP 3, the equilibrium rents for business floorspace $\mathbf{R}$ can be solved with the floorspace demand function (3.5) subject to the stock constraints $\hat{\boldsymbol{B}}$. Similarly, the housing rents $\mathbf{r}$ are solved with the *Marshallian* or *Hicksian* demand function subject to the housing stock constraints $\hat{\boldsymbol{b}}$.

*STEP 5 (Wages).* Given the production price $\boldsymbol{p}$ from STEP 1, the location choices from STEP 2, and the production outputs $\boldsymbol{X}$ from STEP 3, the equilibrium wages $\mathbf{w}$ can be solved with the labour market zero-excess-demand equation.

*STEP 6 (Updating).* Gathering the results of STEP 1 to STEP 5, the algorithm has determined vectors $\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{R}, \boldsymbol{r}$ conditional on transport matrices $\boldsymbol{G}$ and $\boldsymbol{g}$ and all exogenous variables, constraints and parameters. The algorithm will then check whether these updated prices and the associated quantities are converged and whether they simultaneously satisfy all equilibrium conditions to a desired level of accuracy that is discussed below. If not, then the next iteration is started by returning to STEP 1 with these updated vectors. If all equilibrium conditions and converging criteria are satisfied simultaneously, model iteration stops and writes output files.

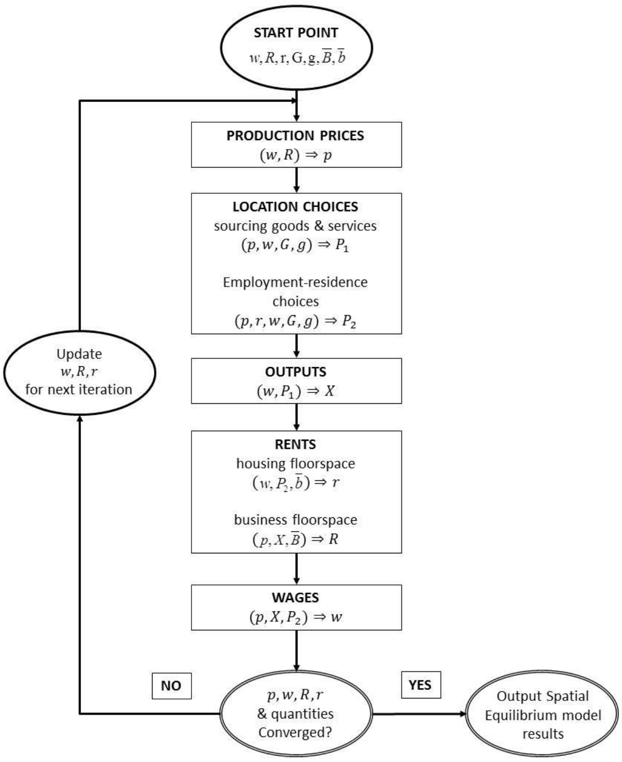


**Figure 22 Solving Algorithm for Spatial Equilibrium Model**

We define the level of converging accuracy by setting a maximum relative error condition. The Spatial Equilibrium model is considered converged in the *n*th iteration when the following inequality condition is satisfied simultaneously for all prices and quantities concerned:

$$\max_{\forall i}\left(\left|\frac{x_{i|n} - x_{i|n-1}}{\frac{1}{2}\left(x_{i|n} + x_{i|n-1}\right)}\right|\right) < ITERTOL \tag{30}$$

where vectors $x_{i|n}$ include zonal prices $\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{R}, \boldsymbol{r}$ and all the associated excess demands in iteration $n$, and $ITERTOL$ is a user-specified maximum iteration tolerance. When the Spatial Equilibrium model is initiated with guesstimated starting values, large relative errors between iterations may occur. As the model approaches the equilibrium solution, the relative errors are expected to reduce gradually, yet not necessarily monotonically.

In order to stabilize the equilibrating process and avoid the model from divergence, we need to define how the variables are updated between iterations. Let $Current(X_n)$ be the variable value in iteration $n$ and $New(X_{n+1})$ be the updated value from the solving algorithm for iteration $n+1$, we set:

$$Current(x_{n+1}) = \varpi(n)New(x_n) + [1 - \varpi(n)]Current(x_n) \tag{31}$$

where coefficient $\varpi(n) \in [0,1]$ is a monotonically increasing function with respect to the iteration number $n \in [1, MAXITER]$. The $\varpi(n)$ function represents a smoothing technique for updating variables between iterations. A smaller step change of $\varpi(n)$ helps to stabilize the equilibrating process but incurs more iterations.

## A3 List of Variables in the Model

| | |
|---|---|
| INDICES FOR DIMENSIONS OF THE MODEL | |
| $\Im$ | Number of core zones |
| $\wp$ | Number of peripheral zones |
| $\mathbb{N} = \Im + \wp$ | Total number of model zones |
| $F$ | Number of social-economic groups |
| $\mathcal{R}$ | Number of industry types |
| $\aleph_1$ | Number of residential floorspace types |
| $\aleph_2$ | Number of business floorspace types |
| $D$ | Exogenous number of annual working days |
| $N = 24D$ | Exogenous total annual time endowment |
| VARIABLES IN SPATIAL EQUILIBRIUM MODEL | |
| $X_{rj}$ | Aggregate production output of industry $r$ in zone $j$ |
| $E_{rj}$ | Constant scalar representing any additional zonal effects on Total Factor Productivity (TFP) |
| $A_{rj}$ | An economic mass function for industry $r$ in zone $j$ that represents the agglomeration effects on TFP |
| $K_r$ | Capital input for industry $r$ |
| $L_{fj}$ | Labour input of type $f$ for industry $r$ in zone $j$ |
| $B_{kj}$ | Business floorspace input of type $k$ for industry $r$ in zone $j$ |
| $Y_{rsj}$ | Intermediate input of type $s$ for industry $r$ in zone $j$ |
| $M_j$ | Economic mass of zone $j$ |
| $S_i$ | Geographic area of zone $j$ |
| $\chi_{fij}$ | Travel disutility function for socio-economic group type $f$ travelling from $i$ to $j$ |
| $p_{rj}$ | Unit production price of industry $r$ in zone $j$ |
| $\rho$ | Real interest rate |
| $w_{fj}$ | Hourly wage of labour type $f$ in zone $j$ |
| $R_{kj}$ | Average rent for business floorspace type $k$ in zone $j$ |
| $p^*_{rs\|j}$ | Average delivered price of intermediate input type $s$ for producing product type $r$ in zone $j$ |
| $U_{fij}$ | Observable utility of resident type $f$ living in zone $i$ and working in zone $j$ |
| $Z_{rz\|fij}$ | Aggregate consumption volume for industry $r$ in zone $z$, given resident type $f$ living in zone $i$ and working in zone $j$ |
| $b_{m\|fij}$ | Consumption volume for housing type $m$ in zone $i$, given resident type $f$ living in zone $i$ and working in zone $j$ |
| $l_{fij}$ | Leisure time of resident type $f$ living in zone $i$ and working in zone $j$ |
| $g_{fiz}$ | Expected one-way monetary cost from $i$ to $z$ for customers type $f$ |
| $G_{fiz}$ | Expected one-way travel time from $i$ to $z$ for customers type $f$ |
| $\mathcal{M}_{fi}$ | Nonwage income of consumer type $f$ in zone $i$ |
| $r_{mi}$ | Housing rent of type $m$ in zone $i$ |
| $\Delta_f$ | Employment status of the consumer type $f$ (For all employed consumers $\Delta_f = 1$; otherwise $\Delta_f = 0$) |
| $p^*_{rz\|fij}$ | Full delivered price of a unit of goods & services type $r$ produced in zone $z$ purchased by consumer type $f$ living in zone $i$ and working in zone $j$ |
| $\Omega_{fij}$ | Full disposable income of the consumer type $(fij)$ net of commuting costs |

| | |
|---|---|
| $\bar{Z}_{r\|fij}$ | Aggregate demand for product type $r$ for consumer type $(fij)$ |
| $\bar{p}_{r\|fij}$ | Probability-weighted average price of product type $r$ faced by consumer type $(fij)$ |
| $N_{fij}$ | Total annual labour working time for labour type $(fij)$ |
| $\widetilde{U}_{fij}$ | Price-based indirect utility of resident type $f$ living in zone $i$ and working in zone $j$ |
| $P_{rz\|fij}$ | Probability of obtaining product type $r$ from zone $z$ to consumer type $f$ living in zone $i$ (and working in zone $j$, if employed) |
| $S_z$ | Size term that corrects for the bias introduced by the uneven sizes of zones in the model |
| $P_{fij}$ | Probability of employed resident type $f$ choosing to live in zone $i$ and work in zone $j$ |
| $\upsilon_{fj}$ | Employment location utility of zone $j$ for labour type $f$ |
| $\upsilon_{fi\|j}$ | Residence location utility of zone $i$ for resident type $f$, given the chosen workplace $j$ |
| $V_{f\|j}$ | *Log-sum* or *inclusive utility* representing the expected utility that employed worker type $f$ in zone $j$ would receive from all residence location choices |
| $\overline{w}_{fi}$ | Average hourly wage of type-$f$ employed residents living in zone $i$ |
| $d_{fij}$ | Travel disutility after Box-Cox transformation for commuter type $f$ travelling from $i$ to $j$ |
| $\hat{b}_{mi}$ | Stock constraints of housing floorspace type $m$ in zone $i$ |
| $\hat{B}_{ki}$ | Stock constraints of business floorspace type $k$ in zone $j$ |
| $H_{fi}$ | Number of type $f$ residents in zone $i$ |
| $\Theta$ | Exogenous nonwage income from other sources |
| $\Xi_{rj}$ | Exogenous net export for industry $r$ in zone $j$ |
| VARIABLES IN RECURSIVE DYNAMIC MODELS | |
| $\hat{B}_{ki}^{t+1}$ | Zonal business floorspace stock of type $k$ at zone $i$ for period $t+1$ |
| $\vec{B}_{k}^{t\|t+1}$ | Regional aggregate stock change of business floorspace type $k$ from period $t$ to $t+1$ |
| $V_{i\|B}$ | Locational utility of zone $j$ for business floorspace growth |
| $\hat{b}_{mi}^{t+1}$ | Zonal housing floorspace stock of type $m$ at zone $i$ for period $t+1$ |
| $\vec{b}_{m}^{t\|t+1}$ | Regional aggregate stock change of housing floorspace type $m$ from period $t$ to $t+1$ |
| $V_{i\|b}$ | Locational utility of zone $j$ for housing floorspace growth |
| $\bar{R}_i^t$ | Zonal average business floorspace rent at zone $i$ for period $t$ |
| $\bar{R}_D^t$ | Municipal/provincial average business floorspace rents at $D$ for period $t$ |
| $\mathcal{D}_i^t$ | Zonal building floorspace density at zone $i$ for period $t$ |
| ב$_{i\|B}$ | Dummy variable indicating zonal policy trend for business floorspace growth |
| $\bar{r}_i^t$ | Zonal average housing floorspace rent at zone $i$ for period $t$ |
| $\bar{r}_D^t$ | Municipal/provincial average housing floorspace rents at $D$ for period $t$ |
| ב$_{i\|b}$ | Dummy variable indicating zonal positive policy trend for housing floorspace growth |
| ג$_{i\|b}$ | Dummy variable indicating zonal negative policy trend for housing floorspace growth |
| $H_{i\|F}^{t+1}$ | Zonal number of non-employed residents in zone $i$ at period $t+1$ |
| $\vec{H}_F^{t\|t+1}$ | Regional aggregate change of non-employed households from period $t$ to $t+1$ |
| $J_{fj}^t$ | Number of labour type $f$ in zone $j$ for period $t$ |

**List of Parameters in the Model**

| PARAMETERS IN SPATIAL EQUILIBRIUM MODEL | |
|---|---|
| $\delta_r$ | Labour cost share |
| $\mu_r$ | Business floorspace cost share |
| $\nu_r$ | Capital cost share |
| $\gamma_{rn}$ | Intermediate cost share |
| $\zeta_r$ | Elasticity of substitution for business floorspace varieties |
| $\theta_r$ | Elasticity of substitution for labour varieties |
| $\sigma_f$ | Elasticity of substitution for housing varieties |
| $a_{f\|\kappa}$ | Coefficient for determining the input-specific parameters for labour varieties |
| $\kappa_{rfj}$ | Input-specific parameters for labour varieties |
| $a_{f\|\iota}$ | Coefficient for determining the input-specific parameters for housing varieties |
| $\xi_{rfz}$ | Input-specific parameters for goods & services varieties |
| $\iota_{mfi}$ | Input-specific parameters for housing varieties |
| $E_j$ | Additional total factor productivity multiplier |
| $\pi$ | Economic mass effects on productivity |
| $c_f$ | Cost for delivering a unit of local services as percentage of commuting trip cost |
| $\alpha_f$ | Utility coefficient for goods & services |
| $\beta_f$ | Utility coefficient for housing |
| $\gamma_f$ | Utility coefficient for leisure time |
| $a_{f\|d}$ | Log-linear travel cost function parameter |
| $\varsigma_f$ | Decay coefficient for value of time (non-commuting travels) |
| $\lambda_{f\|r}$ | Dispersion parameter for sourcing goods & services |
| $\lambda_{f\|J}$ | Dispersion parameter for employment location choices |
| $\lambda_{f\|I}$ | Dispersion parameter for residence location choices |
| $\psi_{iz}, \psi_{fi\|j}, \psi_{fj}$ | Observable non-monetary barriers for spatial interaction |
| $E_{fz}$ | Residual attractiveness for sourcing goods & services |
| $E_{fj}$, $E_{fi\|j}$ | Residual attractiveness for residence-employment location choices |